Appendix B.8: **Elastic Collisions: Analysis of**

Considering an elastic collision where two different masses moving at different velocities both before and after the collision is not easily solved because we have two equations with two unknowns. Consider mass (M_1) moving with initial velocity ($\vec{v}_{1i}$) experience an elastic collision with mass (M_2) moving at velocity ($\vec{v}_{2i}$). The problem is to find the final velocity ($\vec{v}_{1i}$) of mass (M_1) and of mass (M_2) i.e. $\vec{v}_{2f}$, after the collision. To keep the analysis fairly simple, we shall consider that the trajectory of the two masses is along a solitary plane that passes through their center of masses. For any elastic collision the kinetic energy is conserved therefore:

$$(M_1\vec{v}_{1i}^2 + M_2\vec{v}_{2i}^2)/2 = (M_1\vec{v}_{1f}^2 + M_2\vec{v}_{2f}^2)/2 \qquad \text{B.8.1}$$

Multiplying both of eqn B.8.1 sides by 2 gives:

$$M_1\vec{v}_{1i}^2 + M_2\vec{v}_{2i}^2 = M_1\vec{v}_{1f}^2 + M_2\vec{v}_{2f}^2 \qquad \text{B.8.2}$$

Rearranging the variables gives:

$$M_1(\vec{v}_{1i}^2 - \vec{v}_{1f}^2) = M_2(\vec{v}_{2f}^2 - \vec{v}_{2i}^2) \qquad \text{B.8.3}$$

Eqn (B.8.3) can be rewritten in terms of difference of squares:

$$M_1(\vec{v}_{1i} + \vec{v}_{1f})(\vec{v}_{1i} - \vec{v}_{1f}) = M_2(\vec{v}_{2f} + \vec{v}_{2i})(\vec{v}_{2f} - \vec{v}_{2i}) \qquad \text{B.8.4}$$

For any collision conservation of momentum means:

$$M_1\vec{v}_{1i} + M_2\vec{v}_{2i} = M_1\vec{v}_{1f} + M_2\vec{v}_{2f} \qquad \text{B.8.5}$$

Rearranging Eqn B.8.5 gives:

$$M_1(\vec{v}_{1i} - \vec{v}_{1f}) = M_2(\vec{v}_{2f} - \vec{v}_{2i}) \qquad \text{B.8.6}$$

Dividing equation B.8.4 by eqn B.8.6 yields:

$$\vec{v}_{1i} + \vec{v}_{1f} = \vec{v}_{2i} + \vec{v}_{2f} \qquad \text{B.8.7}$$

Rearranging this equation gives an interesting result:

$$\vec{v}_{1i} - \vec{v}_{2i} = \vec{v}_{2f} - \vec{v}_{1f} \qquad \text{B.8.8}$$

As William Layton (physics teacher) explains it; "This equation says the relative velocity before the collision equals minus the relative velocity after the collision. This is always true in all perfectly elastic collisions. A "super" ball should bounce up with the same speed it had just before striking the floor. This equation also makes it easy to solve the original problem". Note: Eqn B.8.8 is in other texts[1]. Dividing eqn B.8.4 by B.8.6 seems strange as we divided conservation of kinetic energy by conservation of momentum. Even so it is doable because of the equalities. Rectifying: Rewrite Eqn B.8.8:

$$\vec{v}_{1i} = \vec{v}_{2f} + \vec{v}_{2i} - \vec{v}_{1f} \qquad \text{B.8.9}$$

Inserting eqn B.8.9 into eqn B.8.5 gives:

$$M_1(\vec{v}_{2f} + \vec{v}_{2i} - \vec{v}_{1f}) + M_2\vec{v}_{2i} = M_1\vec{v}_{1f} + M_2\vec{v}_{2f} \qquad \text{B.8.10}$$

Which can be rewritten:

$$M_1(\vec{v}_{2f} + \vec{v}_{2i} - \vec{v}_{1f} - \vec{v}_{1f}) = M_2(\vec{v}_{2f} - \vec{v}_{2i}) \qquad \text{B.8.11}$$

Collecting the terms gives:

$$M_1(\vec{v}_{2f}+\vec{v}_{2i}-2\vec{v}_{1f})=M_2(\vec{v}_{2f}-\vec{v}_{2i}) \qquad \text{B.8.12}$$

Which can be rewritten as:

$$M_1/M_2=(\vec{v}_{2f}-\vec{v}_{2i})/(\vec{v}_{2f}+\vec{v}_{2i}-2\vec{v}_{1f}) \qquad \text{B.8.13}$$

Eqn (B.8.13) can be rewritten as:

$$M_2/M_1=(\vec{v}_{2f}+\vec{v}_{2i}-2\vec{v}_{1f})/(\vec{v}_{2f}-\vec{v}_{2i}) \qquad \text{B.8.14}$$

Which can be rewritten:

$$M_2/M_1=1-[2\vec{v}_{1f}/(\vec{v}_{2f}-\vec{v}_{2i})] \qquad \text{B.8.15}$$

Certainly eqn 1.10.38 is not some simple discernible equation that easily determines what an elastic collision looks like. There is one solution to (14) that is readily envisioned that being set: $M_1=M_2$, then:

$$(\vec{v}_{2f}+\vec{v}_{2i}-2\vec{v}_{1f})=(\vec{v}_{2f}-\vec{v}_{2i}) \qquad \text{B.8.16}$$

Collecting the variable gives:

$$-2\vec{v}_{1f}=-2\vec{v}_{2i} \qquad \text{B.8.17}$$

Thus the simple solution is:

$$\vec{v}_{1f}=\vec{v}_{2i} \qquad \text{B.8.18}$$

A solution to our problem is that an elastic collision involves two identical masses where all the momentum is transferred from M_1 to $M_{2,}$ with M_2 having an initial velocity of zero. The classic example being in a game of billiards when the cue ball hits another ball through their center of mass, with the only rotation being along the plane through the center of their masses. In this case all the cue ball momentum will be passed onto the other ball. Other solutions remain plausible but these are not simple to visualize.

Inelastic Collision

Certainly inelastic collisions are the norm. Consider a bullet hitting a block of wood where the bullet lodges itself in the wood. Such a collision is not an elastic one hence kinetic energy is not conserved, although energy is conserved with the lost energy being associated with deformation and/or heat. For such an inelastic collision where the two masses become one, then based upon eqn B.8.5:

$$M_1\vec{v}_{1i}+M_2\vec{v}_{2i}=(M_1+M_2)\vec{v}_f \qquad \text{B.8.19}$$

Commentary

In this book, our real concern concerning elastic collisions is one of, is it logical that inter molecular collisions are all elastic as prescribed by traditional kinetic theory, or were we right to question this supposition, as was done in Chapter 7. Nothing was prove but we know most collisions are inelastic.

References:

1. “University Physics” Sears, Francis, Hugh,Young, Zemansky, Mark, Addison Wesley Publishing company . Reading Massechussettes, 1987